electro optical industries inc.

A PROGRAM FOR BLACKBODY RADIATION CALCULATIONS

Planck's Law Programs for the Hewlett Packard model 65 programmable calculator

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INTRODUCTION

This booklet includes 4 programs for the Hewlett Packard Model 65 programmable calculator. These programs transfer the calculation capability of 31 scales of the Electro Optical Industries Inc. blackbody radiation sliderule to the Model 65 but with 5 significant figure accuracy.

A copy of the manual for the blackbody sliderule is included since it defines all terms and equations and provides useful examples of the use of the equations.

PROGRAM TITLE: BLACKBODY RADIATION SLIDERULE

PROGRAM DESCRIPTION, EQUATIONS, VARIABLES:

Planck's expression for hemispherical blackbody flux density radiated in the wavelength interval λ to $\lambda + d\lambda$ is

$$H_\lambda = \frac{c_1}{\lambda^5} \cdot \frac{1}{e^{c_2/\lambda T} - 1} \qquad [\mathrm{W/cm^2 - \mu m}]$$

where T = blackbody temperature (°K)

$c_1 = 37415 \quad \mathrm{W{-}\mu^4/cm^2}$

$c_2 = 14388 \quad \mu{-}°\mathrm{K}$.

The corresponding expression for photon output is

$$Q_\lambda = \frac{c_1'}{\lambda^4} \cdot \frac{1}{e^{c_2/\lambda T} - 1} \qquad [\mathrm{photons/sec{-}cm^2{-}\mu m}]$$

where $c_1' = 188365 \times 10^{18} \quad \mathrm{\mu m^3/sec{-}cm^2}$.

This program computes

$$H_\lambda,\ Q_\lambda,\ \int_0^\lambda H_\lambda d\lambda,\ \int_0^\lambda Q_\lambda d\lambda,\ \int_\lambda^\infty H_\lambda d\lambda,$$

$$\int_\lambda^\infty Q_\lambda d\lambda \text{ given } \lambda \text{ and } T,\ H_{0-\infty} \equiv \int_0^\infty H_\lambda d\lambda,$$

$$Q_{0-\infty} \equiv \int_0^\infty Q_\lambda d\lambda \text{ given } T, \text{ as well as}$$

$T = t_c + 273.15 = 5/9\,(t_f - 32) + 273.15$ given either t_c (Celsius temperature) or t_f (Fahrenheit temperature).

In addition, the program computes RMS Johnson noise

$$V_n = \sqrt{4RkT\Delta f} \qquad [V]$$

across resistance R (Ω) at temperature T (°K) in bandwidth Δf (Hz), given R, T and Δf, as well as photon energy at the maximum of H_λ

$$E_{\lambda m} = 4.96511 \ kT \qquad [ev]$$

given T, where $k = 8.6171 \times 10^{-5}$ ev/°K.

SAMPLE PROBLEMS:

1. For a blackbody at 1500°K, find $H_{0-\infty}$, $Q_{0-\infty}$ and $E_{\lambda m}$.

Solution: $H_{0-\infty} = 2.8704 \times 10^1$ W/cm²

$Q_{0-\infty} = 5.1314 \times 10^{20}$ photons/sec-cm²

$E_{\lambda m} = 6.4177 \times 10^{-1}$ ev

2. Find $\int_{2\mu m}^{10\mu m} H_\lambda \, d\lambda$ and $\int_{25\mu m}^{30\mu m} H_\lambda \, d\lambda$ for a 1000°K blackbody.

Solution: $$\int_{2\mu m}^{10\mu m} H_\lambda \, d\lambda = \int_0^{10\mu m} H_\lambda \, d\lambda - \int_0^{2\mu m} H_\lambda \, d\lambda$$

$$= 5.1829 - .37830 = 4.805 \quad W/cm^2$$

$$\int_{25\mu m}^{30\mu m} H_\lambda \, d\lambda = \int_{25\mu m}^{\infty} H_\lambda \, d\lambda - \int_{30\mu m}^{\infty} H_\lambda \, d\lambda$$

$$= 4.4418 \times 10^{-2} - 2.6698 \times 10^{-2}$$

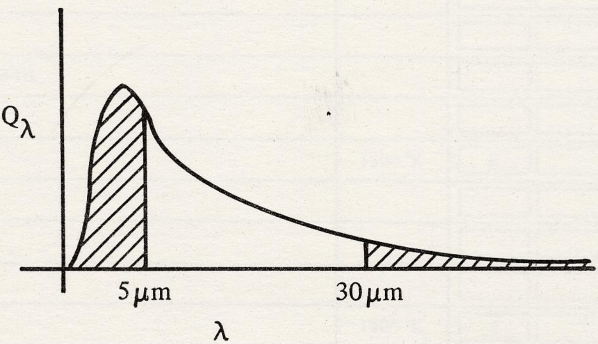
$$= 1.7720 \times 10^{-2} \ W/cm^2$$

3. Calculate V_n across a 10 MΩ resistor at 600°C in a 1.5 Hz band.

Solution: $V_n = 8.5044 \times 10^{-7}\ V_{RMS}$

4. For a 2000°K blackbody, calculate $\int_{5\mu m}^{30\mu m} Q_\lambda \, d\lambda$.

Solution: (refer to diagram)



$$\int_{5\mu m}^{30\mu m} Q_\lambda \, d\lambda = \int_0^\infty Q_\lambda \, d\lambda - \text{total shaded area}$$

$$= Q_{0-\infty} + \left(-\int_0^{5\mu m} Q_\lambda \, d\lambda - \int_{30\mu m}^{\infty} Q_\lambda \, d\lambda \right)$$

$$= 1.2163 \times 10^{21} - 8.9964 \times 10^{20} - 1.3419 \times 10^{19}$$

$$= 3.033 \times 10^{20} \text{ photons/sec-cm}^2$$

REFERENCES:

H.W. Makowski, "A Sliderule for Radiation Calculations," REVIEW OF SCIENTIFIC INSTRUMENTS, 20, 876 (1949)

M. Pivovonsky and M.R. Nagel, TABLES OF BLACKBODY RADIATION FUNCTIONS, Macmillan Co., N.Y. (1961)

SAMPLE PROBLEM I SOLUTION

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1.	Enter program card I				
2.	Compute $H_{O-\infty}$				
	Input T	1500°K	B		
	Read answer				2.8704×10^{1}
					W/cm^2
3.	Enter program card III.				
4.	Compute $Q_{O-\infty}$				
	Input T	1500 °K	B		
	Read answer				5.1314×10^{20}
					photons / sec-cm^2
5.	Compute $E_{\lambda m}$.				
	Input T	1500 °K	C		
	Read answer				6.4177×10^{-1}
					ev

SAMPLE PROBLEM 2 SOLUTION

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1.	Enter program card II				
2.	Compute first integral				
	Input λ_1 and	2 μm	↑		
	T	1000 °K	A		.37830 W/cm^2
	Save answer		STO	4	
	Input λ_2 and	10 μm	↑		
	T	1000 °K	A		5.1829 W/cm^2
	Subtract first answer		RCL	04	
	Read answer		—		4.8046 W/cm^2
3.	Compute second integral				
	Input λ_1 and	25 μm	↑		
	T	1000 °K	A		-4.4418x10^{-2} W/cm^2
	Save answer		STO	4	
	Input λ_2 and	30 μm	↑		
	T	1000 °K	A		-2.6698x10^{-2} W/cm^2
	Subtract first answer		RCL	4	
	Read answer		—		1.7720 x 10^{-2} W/cm^2

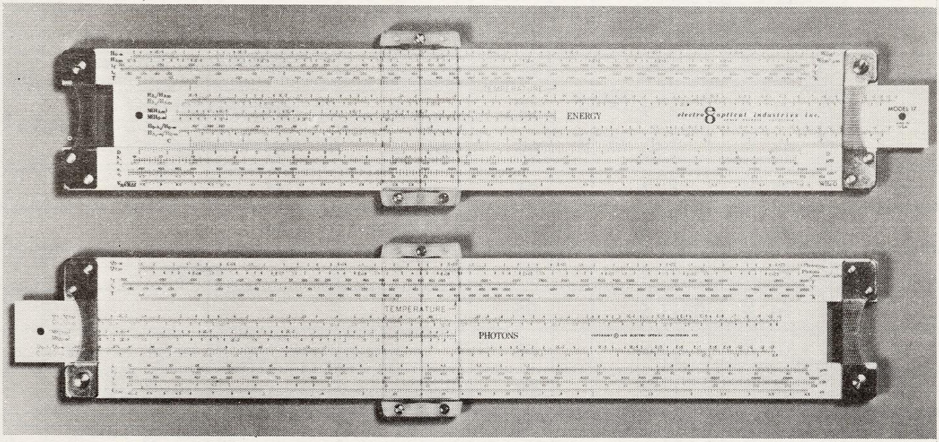
SAMPLE PROBLEM 3 SOLUTION

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1.	Enter program card I				
2.	Convert t_c to T	600 °C	D		873.15 °K
3.	Input R	$10^7\,\Omega$	↑		
4.	Input Δf, compute V_n, read answer	1.5	C		8.5044×10^{-7} V_{RMS}

SAMPLE PROBLEM 4 SOLUTION

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1.	Enter program card IV				
	Compute last 2 integrals				
	Input λ_1 and	5 μm	↑		
	T	2000 °K	A		8.9964×10^{20}
	Save answer		STO	04	photons/ sec-cm^2
	Input λ_2 and	30 μm	↑		
	T	2000 °K	A		-1.3419×10^{19}
					photons/ sec-cm^2
	Subtract first answer		RCL	04	
			−		-9.1305×10^{20}
	Save answer		STO	04	photons/ sec-cm^2
2.	Enter program card III				
	Compute first integral				
	Input T	2000 °K	B		1.2163×10^{21}
	Add first answer		RCL	04	photons/ sec-cm^2
	Read final answer		+		3.0327×10^{20}
					photons/ sec-cm^2

HEWLETT-PACKARD'S HP65
PROGRAMMABLE
POCKET-SIZED CALCULATOR



ELECTRO OPTICAL INDUSTRIES'

MODEL 17
RADIATION SLIDERULE

TITLE **BLACKBODY RADIATION SLIDERULE I**

SWITCH TO W/PRGM. PRESS [f] [PRGM] TO CLEAR MEMORY

KEY ENTRY	CODE SHOWN	KEY ENTRY	CODE SHOWN	KEY ENTRY	CODE SHOWN
LBL	23	B	12	LBL	23
A	11	↑	41	D	14
g x↕y	35 07	X	71	2	02
↑	41	↑	41	7	07
↑	41	X	71	3	03
gR↑	35 09	1	01	•	83
X	71	7	07	1	01
1	01	6	06	5	05
4	04	3	03	+	61
3	03	7	07	DSP	21
8	08	EEX	43	•	83
8	08	7	07	3	03
g x↕y	35 07	÷	81	RTN	24
÷	81	DSP	21	LBL	23
f^{-1}	32	4	04	E	15
LN	07	RTN	24	3	03
1	01	LBL	23	2	02
–	51	C	13	–	51
X	71	X	71	5	05
X	71	X	71	X	71
X	71	f	31	9	09
X	71	$\sqrt{x}$	09	÷	81
X	71	1	01	D	14
3	03	3	03	RTN	24
7	07	4	04		
4	04	5	05		
1	01	6	06		
5	05	9	09		
g x↕y	35 07	EEX	43		
÷	81	6	06		
DSP	21	÷	81		
4	04	DSP	21		
RTN	24	4	04		
LBL	23	RTN	24		

REGISTERS

R_1

R_2

R_3

R_4

R_5

R_6

R_7

R_8

R_9

LABELS

- A H_λ
- B $H_{0-\infty}$
- C V_n
- D $t_c \rightarrow T$
- E $t_f \rightarrow T$

H_λ $H_{0-\infty}$ V_n $t_c \rightarrow T$ $t_f \rightarrow T$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1.	Enter program				
2.	To calculate H_λ				
	input wavelength λ and	λ (μm)	↑		
	blackbody temperature T	T(°K)	A		
	read H_λ				H_λ
					(W/cm^2-μm)
3.	To calculate $H_{0-\infty}$				
	input blackbody temperature T	T(°K)	B		
	read $H_{0-\infty}$				$H_{0-\infty}$
					(W/cm^2)
4.	To calculate V_n				
	input temperature T	T(°K)	↑		
	input resistance R	R(Ω)	↑		
	input bandwidth Δf	Δf (Hz)	C		
	read V_n				V_n (V)
5.	To calculate T input either				
	Celsius temperature t_c or	t_c (°C)	D		
	Fahrenheit temperature t_f	t_f (°F)	E		
	read T				T(°K)
	NOTES				
1.	Memory register $R_1 - R_9$ are unaffected				
	by these programs, and thus may be used to store				
	intermediate results.				

TITLE BLACKBODY RADIATION SLIDERULE II

SWITCH TO W/PRGM. PRESS [f] [PRGM] TO CLEAR MEMORY

KEY ENTRY	CODE SHOWN
0	00
STO 3	33 03
STO 8	33 08
+	61
STO 2	33 02
X	71
1	01
4	04
3	03
8	08
8	08
gx ⇄ y	35 07
÷	81
STO 1	33 01
•	83
8	08
gx ≤ y	35 22
GTO	22
1	01
RCL 1	34 01
↑	41
X	71
1	01
0	00
5	05
÷	81
–	51
X	71
6	06
–	51
X	71
1	01
6	06
+	61

KEY ENTRY	CODE SHOWN
X	71
X	71
X	71
4	04
8	08
÷	81
GTO	22
2	02
LBL	23
1	01
g	35
DSZ	83
RCL 8	34 08
RCL 1	34 01
X	71
↑	41
↑	41
↑	41
3	03
–	51
X	71
6	06
+	61
X	71
6	06
–	51
gR ↑	35 09
f^{-1}	32
LN	07
X	71
RCL 8	34 08
↑	41
X	71
↑	41

KEY ENTRY	CODE SHOWN
X	71
÷	81
STO	33
+	61
3	03
EEX	43
6	06
X	71
RCL 3	34 03
gx > y	35 24
GTO	22
1	01
LBL	23
2	02
CHS	42
RCL 2	34 02
↑	41
X	71
↑	41
X	71
X	71
1	01
1	01
4	04
5	05
4	04
EEX	43
8	08
÷	81
DSP	21
4	04
R/S	84

REGISTERS	
R_1	$c_2/\lambda T$
R_2	T
R_3	$\sum$
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	

LABELS	
A	$\int H_\lambda \, d\lambda$
B	
C	
D	
E	

$\int H_\lambda \, d\lambda$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1.	Enter program				
2.	Input wavelength λ and	$\lambda (\mu m)$	↑		
	blackbody temperature T	T(°K)			
3.	Compute $\int H_\lambda d\lambda$. If positive,		A		$\int H_\lambda d\lambda$
	answer is $\int_0^\lambda H_\lambda d\lambda$. If negative,				(W/cm ^{2})
	answer is $- \int_\lambda^\infty H_\lambda d\lambda$.				
	NOTES				
1.	This program automatically chooses which integral to				
	compute so as to provide in all cases				
	a 5-significant-figure answer.				
2.	Since memory registers $R_4 - R_7$ are not affected by this				
	program, they may be used to store intermediate results.				

TITLE BLACKBODY RADIATION SLIDERULE III

SWITCH TO W/PRGM. PRESS [f] [PRGM] TO CLEAR MEMORY

KEY ENTRY	CODE SHOWN
LBL	23
A	11
gx ⇄ y	35 07
↑	41
↑	41
gR ↑	35 09
X	71
1	01
4	04
3	03
8	08
8	08
gx ⇄ y	35 07
÷	81
f^{-1}	32
LN	07
1	01
–	51
X	71
X	71
X	71
X	71
1	01
8	08
8	08
3	03
6	06
5	05
EEX	43
1	01
8	08
gx ⇄ y	35 07
÷	81
DSP	21

KEY ENTRY	CODE SHOWN
4	04
RTN	24
LBL	23
B	12
↑	41
↑	41
X	71
X	71
1	01
5	05
2	02
0	00
4	04
EEX	43
7	07
X	71
DSP	21
4	04
RTN	24
LBL	23
C	13
2	02
3	03
3	03
7	07
•	83
3	03
÷	81
DSP	21
4	04
RTN	24
LBL	23
D	14
2	02

KEY ENTRY	CODE SHOWN
7	07
3	03
•	83
1	01
5	05
+	61
DSP	21
•	83
3	03
RTN	24
LBL	23
E	15
3	03
2	02
–	51
5	05
X	71
9	09
÷	81
D	14
RTN	24

REGISTERS
R_1
R_2
R_3
R_4
R_5
R_6
R_7
R_8
R_9

LABELS	
A	Q_λ
B	$Q_{0-\infty}$
C	$E\ \lambda_m$
D	$t_c \to T$
E	$t_f \to T$

Q_λ $Q_{0-\infty}$ $E_{\lambda m}$ $t_c \rightarrow T$ $t_f \rightarrow T$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1.	Enter program				
2.	To calculate Q_λ				
	input wavelength λ and	$\lambda (\mu m)$	↑		
	blackbody temperature T	T(°K)	A		
	read Q_λ				Q_λ
					(photons/ sec-cm^2-μm)
3.	To calculate $Q_{0-\infty}$				
	input blackbody temperature T	T(°K)	B		
	read $Q_{0-\infty}$				$Q_{0-\infty}$
					(photons/ sec-cm^2)
4.	To calculate $E_{\lambda m}$				
	input blackbody temperature T	T(°K)	C		
	read $E_{\lambda m}$				$E_{\lambda m}$ (ev)
5.	To calculate T input either				
	Celsius temperature t_c, or	t_c (°C)	D		
	Fahrenheit temperature t_f	t_f (°F)	E		
	read T				T(°K)
	NOTES				
1.	Memory registers $R_1 - R_9$ are				
	unaffected by these programs, and				
	thus may be used to store intermediate results.				

TITLE **BLACKBODY RADIATION SLIDERULE IV**

SWITCH TO W/PRGM. PRESS [f] [PRGM] TO CLEAR MEMORY

KEY ENTRY	CODE SHOWN	KEY ENTRY	CODE SHOWN	KEY ENTRY	CODE SHOWN
0	00	2	02	÷	81
STO 3	33 03	4	04	STO	33
STO 8	33 08	−	51	+	61
+	61	X	71	3	03
STO 2	33 02	X	71	EEX	43
X	71	4	04	6	06
1	01	8	08	X	71
4	04	÷	81	RCL 3	34 03
3	03	GTO	22	gx > y	35 24
8	08	2	02	GTO	22
8	08	LBL	23	1	01
gx ⇄ y	35 07	1	01	CHS	42
÷	81	g	35	LBL	23
STO 1	33 01	DSZ	83	2	02
•	83	RCL 8	34 08	RCL 2	34 02
8	08	RCL 1	34 01	↑	41
7	07	X	71	↑	41
gx ≤ y	35 22	↑	41	X	71
GTO	22	↑	41	X	71
1	01	1	01	X	71
RCL 1	34 01	−	51	6	06
↑	41	↑	41	3	03
↑	41	X	71	2	02
↑	41	1	01	4	04
X	71	+	61	2	02
9	09	gx ⇄ y	35 07	EEX	43
0	00	f^{-1}	32	6	06
÷	81	LN	07	X	71
1	01	X	71	DSP	21
−	51	RCL 8	34 08	4	04
X	71	↑	41	R/S	84
8	08	↑	41	gNOP	35 01
+	61	X	71		
X	71	X	71		

REGISTERS

- R_1 $c_2/\lambda T$
- R_2 T
- R_3 $\sum$
- R_4
- R_5
- R_6
- R_7
- R_8 n
- R_9 used

LABELS

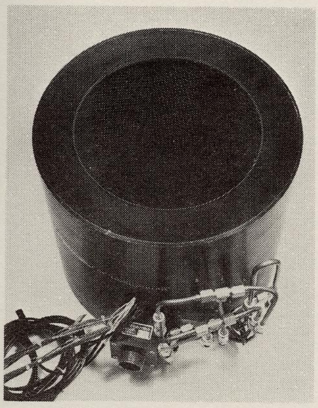
- A $\int Q_\lambda d\lambda$
- B
- C
- D
- E

$\int Q_\lambda d\lambda$

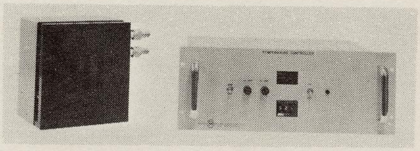
STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1.	Enter program				
2.	Input wavelength λ and	$\lambda(\mu m)$	↑		
	blackbody temperature T	T(°K)			
3.	Compute $\int Q_\lambda d\lambda$. If positive,		A		$\int Q_\lambda d\lambda$
	answer is $\int_0^\lambda Q_\lambda d\lambda$. If negative,				(photons/ sec–cm^2)
	answer is $-\int_\lambda^\infty Q_\lambda d\lambda$.				
	NOTES				
1.	This program automatically chooses which integral				
	to compute, so as to provide in all cases a 5-significant-				
	figure answer.				
2.	Since memory registers R_4 - R_7 are not affected by this				
	program, they may be used to store intermediate results.				

NOTES:

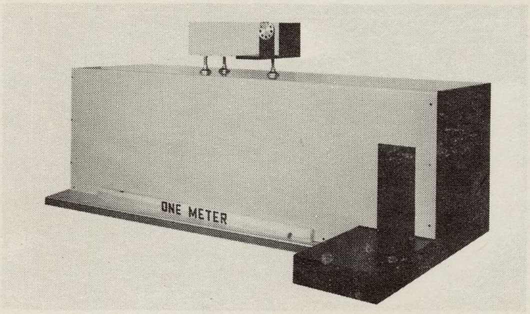
NOTES:



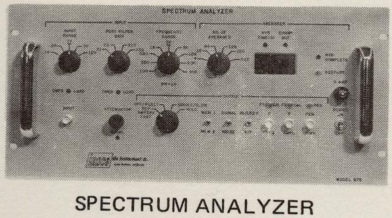
CRYOGENIC VACUUM
BLACKBODIES



THERMOELECTRIC
DIFFERENTIAL
BLACKBODIES

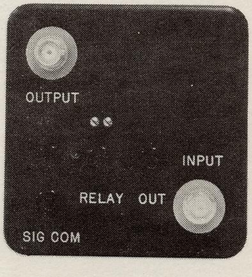


COLLIMATORS
1 inch to 12 inches

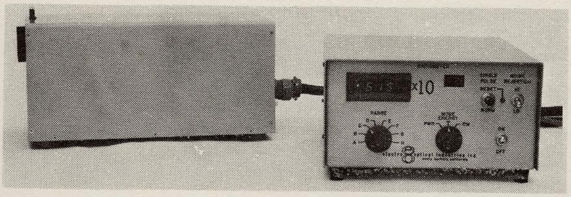


SPECTRUM ANALYZER
1Hz to 50kHz

PREAMPLIFIER
low impedance



SPECTRORADIOMETERS



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